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Bianchi type-1 string cosmology with a scalar field

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Abstract . The Bianchi type-1 anisotropic cosmological model containing a self – interacting scalar field with an exponential potential of the form $V(\Phi) = e^{k\Phi}$ in the context of cosmic strings have been studied Physical features of this model are briefly discussed

Keywords Strings, Scalar field, Bianchi-1 cosmology

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1. Introduction

The study of 'cosmic strings' has attracted a lot of attention, as they are believed to give a compelling and plausible description of the early stage of the universe. Gauge theories with spontaneous symmetry breaking in elementary particle physics have given rise to an intensive study of cosmic strings. It appears that after the big-bang the universe may have experienced a number of phase transitions. These phase transitions can produce vacuum domain structures such as domain walls, strings and monopoles [1]. Of all these structures, cosmic strings are of considerable interest. Letelier [2] studied a model of massive strings in the context of general relativity. He observed that the early stage of the universe was dominated by massive strings and during the evolution of the universe; the strings disappear leaving only particles. Self-interacting scalar fields play a central role in studies of inflationary cosmology [3-6]. It is observed that the scalar fields which drives inflation for F R W models will not make the anisotropic cosmological models inflate [3].

The objective of the paper is to study cosmic strings along with a self-interacting scalar -field with an exponential potential of the form $V(\Phi) \sim e^{k\Phi}$, where k is a non-negative constant to Bianchi-type-1 cosmology . This model has been chosen because

Bianchi type −1 cosmology is supposed to be a reasonable representation of the early universe . The scale factors are assumed to have power- law behaviour. This would give power- law inflation in an F R W model .We here show that unlike F R W model, the power- law solutions for the anisotropic model (here, we consider) shows neither inflation nor isotropization and further we propose to show that during the evolution of the universe the strings disappear leaving only particles and ultimately the scalar field helps the formation of galaxy.

2. Einstein equations coupled to a system of strings and a scalar field

The Einstein equations for a system of strings with scalar field are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \rho u_\mu u_\nu - \lambda x_\mu x_\nu + \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left\{ \frac{1}{2} g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + v(\phi) \right\}. \tag{2.1}$$

Here ρ is the rest energy density of the system of strings with massive particles attached to them . $\rho \equiv \rho_p + \lambda$, ρ_p being the rest energy density of particles and λ is tension density of the system of strings. The vector u^μ describes the system four velocity and x^μ represents a direction of anisotropy; ϕ is a scalar field and $V(\phi)$ is a potential function. We have here [2]

$$u^\mu u_\mu = -x^\mu x_\mu = 1 \text{ and } u^\mu x_\mu = 0. \tag{2.2}$$

The contracted Bianchi identity for (2.1) is equivalent to

$$\nabla_\mu \left(\rho u^\mu\right) - \lambda\, x'^\nu u_\nu = 0 \tag{2.3}$$

$$\nabla_\mu \left(\lambda x^\mu\right) - \rho \dot{u}^\nu x_\nu = 0 \tag{2.4}$$

$$H_\upsilon^\mu \left(\rho\, \dot{u}^\nu - \lambda\, x'^\nu\right) = 0 \tag{2.5}$$

where $x'^\nu = x^\mu \nabla_\mu (x^\nu)$ and $\dot{u}^\nu = u^\mu \nabla_\mu (u^\nu)$.

And H_υ^μ is the 'Projection operator' that projects in the direction that are perpendicular to both x^μ and u^μ. The scalar field ϕ satisfies the equation

$$\partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left\{ \frac{1}{2} g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + V(\phi) \right\} \Big] ;_{\upsilon} = 0 \tag{2.6}$$

Equation (2 3), (2 4) and (2 5) are the evolutions for the system of strings These equations are also the integrability conditions for (2 1) The deceleration parameter q is given by

$$q = -3\theta^{2}\left[\theta + \frac{1}{3}\theta^{2}\right] \tag{2 7}$$

where $\theta - u^{\alpha}{}_{\alpha}$, volume expansion of the fluid and ,' means the covariant derivative

3 Field equations and their integrals

The line element for Bianchi type −1 universe is

$$ds^{2} = dt^{2} - a_1^{2}(t)\,(dx^{1})^{2} - a_2^{2}(t)\,(dx^{2})^{2} - a_3^{2}(t)\,(dx^{3})^{2} \tag{3 1}$$

where a_1, a_2, a_3 are functions of t only

From equations (2 1), (2 2), and (3 1), we may write

$$u^{\mu} = u_{\mu} = (1,0,0,0) \tag{3 2}$$

And x^{μ} must be taken along any of the three directions $\frac{\partial}{\partial x^{1}}, \frac{\partial}{\partial x^{2}}, \frac{\partial}{\partial x^{3}}$

Without losing generality, we choose x^{μ} parallel to $\frac{\partial}{\partial x^{1}}$, so that

$$x^{\mu} = \left(0,\, a_1^{\,1},\, 0,\, 0,\right) \tag{3 3}$$

Applying equations (3 1), (3 2), and (3 3) in the Bianchi identities (2 3), (2 4) (2 5) and (2 6) one gets

$$\rho + (\rho - \lambda)\frac{a_1}{a_1} + \rho\left(\frac{a_2}{a_2} + \frac{a_3}{a_3}\right) = 0 \tag{3 4}$$

and

$$\phi\,\phi + \left(\frac{a_1}{a_1} + \frac{a_2}{a_2} + \frac{a_3}{a_3}\right)\phi^{2} - v(\phi) = 0 \tag{3 5}$$

Here the dot ( ) denotes the derivative with respect to t only Also it is to be noted that as a consequence of the Einstein equations ρ and λ are functions of t only

The Einstein field equations (2 1) for the metric (3 1) are

$$\frac{a_1}{a_1}\frac{a_2}{a_2} + \frac{a_2}{a_2}\frac{a_3}{a_3} + \frac{a_3}{a_3}\frac{a_1}{a_1} = \rho + \left(\frac{1}{2}\phi^{2} - V(\phi)\right) \tag{3 6}$$

$$\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_2\dot{a}_3}{a_2 a_3} = \lambda-\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right) \tag{3.7}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1 a_3} = -\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right) \tag{3.8}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} = -\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right). \tag{3.9}$$

The solutions of the equations (3.5) to (3.9) are

$$a_1 = t^m,\ m>0 \tag{3.10}$$

$$a_2 = a_3 = t^{2m-1}\left(m>\frac{1}{2}\right) \tag{3.11}$$

$$\lambda = (m-1)\,(5m-3)\,t^{-2} \tag{3.12}$$

$$\rho = \frac{m(m-1)\,(5m-3)}{5m-4}\,t^{-2} \tag{3.13}$$

$$\phi = \sqrt{\frac{2(7m^2-8m+2)}{5m-4}}\,\log t \tag{3.14}$$

$$V(\phi) = -\left[\frac{(5m-3)(7m^2-8m+2)}{(5m-4)}\right]t^{-2} \tag{3.15}$$

$$\rho_p = -\frac{4(m-1)^2(5m-3)}{(5m-4)}\,t^{-2} \tag{3.16}$$

$$q = -\frac{5(5m-2)^3\,(m-1)}{t^4}. \tag{3.17}$$

The solutions (3.10) to (3.16) identically satisfy the equation (3.4).

4. Discussion

From solutions (3.10) to (3.17), it is observed that for m in the interval $3/5 < m < 4/5$, $\lambda < 0$, $\rho > 0$, $\rho_p > 0$, $q > 0$ and ϕ is real.

Also we have $$\frac{\rho_p}{|\lambda|} = \frac{4(m-1)}{5m-4} > 1 \text{ for } \frac{3}{5} < m < \frac{4}{5} \tag{4 1}$$

Thus from (4.1), we conclude that throughout the process of evolution, the universe is dominated by massive strings for $3/5 < m < 4/5$ Since q is positive for $3\ 5 < m < 4\ 5$, there is no inflation at any stage Further, $a_1 \neq a_2 = a_3$, so the universe is anisotropic throughout the process of evolution For $m = \left(4 + \sqrt{2}\right)\ 7$, ϕ and V vanish and the expansion of the universe is guided by a particular value of m

We have considered Bianchi type-1 anisotropic model universe filled with self- interacting scalar fields In the solution of the Einstein equations, we have assumed that the scale factors exhibit powers of time enabling us to find exact solutions with the scalar field having an exponential potential It is observed that in the era of cosmic strings, the scalar field which led to inflation and isotropy in F R W models, do not lead to one in which anisotropy is present from the beginning We have seen in the solution of Bianchi type – 1 model that the scalar fields which, are used for F RW model, fail to inflate and isotropize in the early stage of the universe It is also interesting to note that due to the presence of scalar field, the power index '*m*' of the metric co-efficient has a range of values and when ϕ vanish '*m*' has a fixed value Thus we may conclude that the scalar field (ϕ) plays a significant role in the process of evolution of the universe

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